Indian J. Phys. **44**, 258–263 (1970)

A coincidence method for absolute measurement of incident energy of electrons

By P. K. Bhattacharya* and M. R. Bhiday

Department of Applied Physics, S G.S. Institute of Technology and Science
17, Park Road, Indore-3, M.P.

(*Received* 3 *April*, 1970)

A coincidence method is devised for absolute measurement of energy of electrons upto 5 MeV. The method makes use of the fact that the angle between the scattered and recoil particles is dependent on the relativistic energy of the incident electrons. Using fast coincidence techniques and fast multiparameter analysis system in conjunction with fast directional scintillators like 'Phoswich' to detect the two particles, the method can be made accurate to $\pm 1\%$. Employing thin foil targets, multiple scattering ($Z \simeq 6$) and hence, the angular spread in the coincidence peak becomes small. Thus, it is possible to measure the beam energy at the target simultaneously with the scattering cross-section without the necessity of making any change in the experimental set up.

Introduction

Electron scattering experiments require an accurate and absolute measurement of energy of incident particles at the target. In addition, it is necessary to monitor the beam energy continuously during the course of an actual run. By using internal conversion electrons, it is possible to achieve the energy measurement with precision (Siegbahn 1965), using well known standard lines. We could have easily decided upon a suitable definition of the position of the line, if the conversion electrons emanated from an 'ideal source', without any inherent errors like Landau energy loss in the foil. However, in most of these cases, conversion lines from different atomic shells have different shapes especially for low energy lines than for high energy lines, thereby limiting the accuracy of tracing the centroid. Thus, even after knowing the energy standards with a high degree of accuracy the method proves to be only relative. Threshold Čerenkov detectors (Bhiday *et al* 1958) have proved to be good alternatives for absolute measurement of the energy of electrons near 5 MeV and have shown their use, also in applications of flux measurements. Unfortunately, the energy resolution for such counters becomes unsatisfactory below 2 MeV due to larger energy losses, while about 5 MeV there is a little change in beta (v/c) values of electrons with energy, rendering the Čerenkov method ineffective. The problem of absolute measurement of electron energies

*Computer programming and associated calculations were done while working in CERN as a Visiting-Scientist with Neutrino Beam Studies Group in The Directorate of Nuclear Physics Apparatus, 1211, Geneva 23, Switzerland.

in the range 2 to 5 MeV becomes important especially when measurements of electron-nuclear or electron-electron scattering cross-sections are to be undertaken. Such experiments are in progress in this laboratory with the help of a 5 MeV Betatron now made available to us to study the effects of (1) exchange terms (Bhabha 1936, Möller 1932), (2) atomic screening constants (Dougal 196[illegible]; Koch *et al* 1964), and multiple scattering in various foils

This paper deals with the design of a coincidence method for absolute beam energy measurement and its monitoring and shows feasibility of determination of scattering cross-section in the same experimental set up for the cases of electron-electron, positron-electron and electron-nuclear scattering

2. Theory of the Method

We shall use the kinematic relations of collisions between two particles, which are relativistic and assume that an incident particle of mass m_1 collides with a free particle of mass m_2, where, subscripts 0, 1 and 2 hereafter denote quantities related with the incident, scattered and the recoil particles, while subscript c relates to the corresponding values in barycentric coordinates (*i.e.*, with reference to centre of mass system), shown in figure 1.

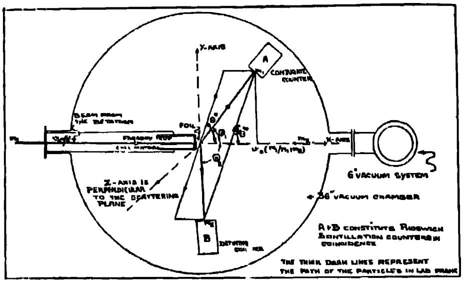

Figure 1. Collision kinematics for energy equation.

The momentum and energy relationships form a Lorentz four vector and are held by the equations,

$$cp_c = \gamma_c(cp_x - \beta_c E) \quad \cdots \quad (1)$$

$$cp_x = \gamma_c(cp_c + \beta_c E_c) \quad \cdots \quad (2)$$

$$E_c = \gamma_c(E - \beta_c cp_x) \quad \cdots \quad (3)$$

$$E = \gamma_c(E_c + \beta_c cp_c) \quad \cdots \quad (4)$$

$$cp_o = cp_{y,z} \quad \cdots \quad (5)$$

and

$$c^2p^2 - E^2 = Q \quad \cdots \quad (6)$$

whence, Q is invariant to the frame of reference.

These relations connect x, y, and z components of the momentum p and energy E, where $p_\perp$, p_y, p_z along with E/c^2 form a four vector. Applying conservation of momentum to the scattered and recoil particles (in C.M. system) we get,

$$m_1^2(\gamma_{c1}^2-1) = m_2^2(\gamma_c^2-1) \qquad \dots \quad (7)$$

where γ_{c1} and γ_c denote the values of $\gamma = 1/(1-v^2/c^2)^{\frac{1}{2}}$ of m_1 and m_2. On putting γ_{c1} and γ_c in the relations for momentum of m_1 in barycentric and laboratory (LAB) system and simplifying to evaluate them by substituting $m_1/m_2 = k$, we get,

$$(\gamma_{c1}^2-1)^{\frac{1}{2}}(1+\gamma_0 k) = \gamma_c(\gamma_0^2-1)^{\frac{1}{2}} \qquad \dots \quad (8)$$

On further simplification we obtain,

$$\gamma_{c1} = (\gamma_0+k)/(1+2k\gamma_0+k^2)^{\frac{1}{2}} \qquad \dots \quad (9)$$

$$\gamma_c = (1+\gamma_0 k)/(1+2k\gamma_0+k^2)^{\frac{1}{2}} \qquad \dots \quad (10)$$

From relations (9) and (10) the velocities of m_1 and m_2 can be found reducing to a simple expression,

$$\gamma_c = \gamma_{c1} = [(1+\gamma_0)/2]^{\frac{1}{2}} \qquad \dots \quad (11)$$

in the particular case of $e-e$ or $p-e$ scattering, as m_1/m_2 or k equals unity.

Using equation (2) and combining (5) and (6), one gets resolved components of momenta in x and y directions to find ultimately the relationships between angles in the LAB and C.M. systems. We get therefrom,

$$\tan\Theta_2 = -\cot(\theta^*/2)/\gamma_c \qquad \dots \quad (12)$$

and

$$\tan\Theta_1 = \sin\theta^*/\gamma_c(\cos\theta^*+k(\gamma_{c1}/\gamma_c)) \qquad \dots \quad (13)$$

The negative sign in equation (12) indicates that m_2 recoils on the other side of the collision axis, which can be accurately defined by collimator slits for the Betatron beam which is limited to a spot of a few mm.

To get relationships of LAB angle Θ_1 and Θ_2 in terms of θ^*, the centre of mass angle of scatter of m_1, we solve equation (13), a quadratic in $\cos\theta^*$ using equation (7), and find,

$$\cos\theta^* = \frac{-\gamma_{c1}\gamma_c k\tan^2\Theta_1 \pm ((1-k^2)\tan^2\Theta_1+1)^{\frac{1}{2}}}{\gamma_c^2\tan^2\Theta_1+1} \qquad \dots \quad (14)$$

The foregoing analysis, if applied to a non-relativistic case, makes the radical zero (or $\tan^2\Theta_1 = 1/(k^2-1)$) and fixes an upper limit to the scattering angle to 90^0, as $\sin\Theta_1 = 1/k = 1$ for identical particle scatters. For $k < 1$ the roots may be derived using equation (14), whence for $\Theta_1 > 90^\circ$ more negative root should be chosen and for $\Theta_1 < 90^0$ only the more positive root must be taken. On eliminating θ^* from equations (12) and (13) for identical particle case, $\gamma_{c1} = \gamma_c$, we obtain an absolute relation for computation of energy as,

$$\tan\Theta_1\tan\Theta_2 = -\gamma_c^{-2} = -2/(1+\gamma_0) \qquad \dots \quad (15)$$

Assuming $\Phi = \Theta_1 + \Theta_2$ as the angle between the scattered and recoil particles, we find by proper substitution for $\tan \Theta_1$ and $\tan \Theta_2$,

$$\tan \Phi = 2/(\beta_c{}^2 \gamma_c \sin \theta^*) \qquad \dots \quad (16)$$

It can be seen from figure 2, that for $\beta_c = 0.96$ the angle Φ rapidly falls down to $32°$ when $\theta^* = 90°$, showing a gradual fall in the angle of scattering between the particles as the incident particle attains relativistic velocity.

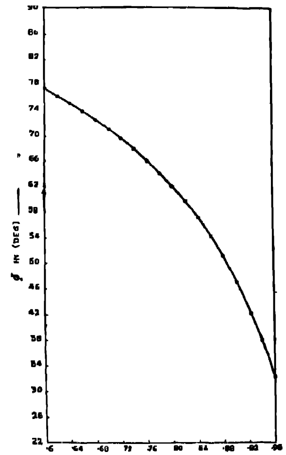

FIGURE 2. Variation of angle Φ as a function of β_c for $\theta^* = 90°$.

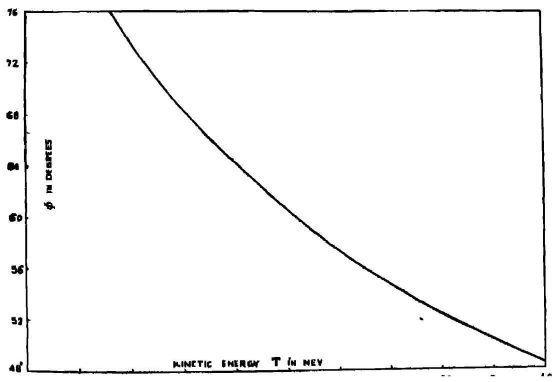

FIGURE 3. Variation of angle Φ between the defining and the conjugate counter with energy T of the incident electrons at the target.

The computations for angle Φ from equation (16) using symmetrical detector situation, are plotted in figure 3 for various kinetic energies. Thus, the energy equation can be used to check the accuracy of the beam energy at various stages during the experimental run by devising a multiparameter system in conjunction with a fast coincidence system to measure the coincidence counting rate by fixing the defining counter B and moving the conjugate A to a position, where a maximum in coincidence counting rate is found. An accuracy of $\pm 3\%$ can be achieved for incident energies of the order of 2 MeV for an error of $\pm 0.5^0$ in the measurement of Φ. However, it can be seen that the accuracy improves by $\pm 0.6\%$ at higher energies. This simple method of measuring coincidence in the scattered and recoil particles not only requires a thin foil of low atomic number (terylene, nylon or melinex) as a target but also depends on (1) true $e-e$ or $p-e$ scattering, (2) the statistical accuracy of the experiment (*i e.* true/chance coincidence rate) and (3) energy spread and geometrical resolution of the beam and the detector system.

3. Factors Influencing Particle Energy

Combination of fast and a slow phosphor as 'Phoswich' (Wilkinson 1952) can be used to detect the particles with highly directional properties. Fast phosphors like NE 111, (full width at half maximum = 1.54 ns) plastic scintillators can be combined with slow phosphors like CsI (Tl) to realise a very effective and directional counting by a single photomultiplier. The fast phosphor would give a sharp spike on the leading edge of a gradually falling slope of the pulse due to slow phosphor when electrons pass both the phosphors. This type of 'Phoswich' would, therefore, not only act as an anti-coincidence shield but also allow coupling of a computer system to the coincidence counters (Wiegand 1959), if an appropriate design of a collimator system is put in front of the defining and conjugate counters looking for a compromise between resolution and sensitivity (Hušàk and Peřinovà 1969). Further, by use of such a system no limitations are put on the solid angle of acceptance, even if one prefers to use surface barrier type solid state detectors having a depletion depth suitable to the maximum incident energy. Using an intense beam of electrons (from a 5 MeV Betatron) and thin foil targets, multiple scattering errors become small and take place only in the target plane so that its effects can be readily allowed for by computing the rms angle of scatter. The angular spread in the coincidence peak thus produced, would not be so large as to affect the energy measurement

4. Accuracy Parameters

We can evaluate the accuracy parameter for the system assuming a particular case of symmetry, whence, $\Theta_1 = \Theta_2$, or $\tan \Phi = 2 \tan \Theta_2/1-\tan^2\Theta_2 = \tan 2\Theta_2$ and similarly $\cos 2\Theta_2 = 1-\tan^2\Theta_2/(1+\tan^2\Theta_2)$. From equation (15) we get, on simplification $\cos 2\Theta_2 = 1-(1-\beta_0^2)^{\frac{1}{2}}/(1+3(1-\beta_0^2)^{\frac{1}{2}}) = A$ say, which on differentiation gives,

$$dA/d\Theta_2 = -2 \sin 2\Theta_2 = -4\beta_0(1-\beta_0^2)^{-\frac{1}{2}}d\beta_0/\{1+3(1-\beta_0^2)^{\frac{1}{2}}\}^2 d\Theta_2$$

or

$$d\beta_0 = \{1+3(1-\beta_0^2)^{\frac{1}{2}}\}^2 \sin 2\Theta_2 d\Theta_2/2\beta_0(1-\beta_0^2)^{-\frac{1}{2}}$$

One can calculate the error in the measurement of the incident beam energy to a near approximation, using the relation $dT = 0.51(1-\beta_0^2)^{-3/2}\beta_0 d\beta_0$ for a known inaccuracy of the measurement of angle Θ_2. Thus, measurements of angle Φ with an accuracy of $\pm 0°.1$ makes the system accurate to $\pm 1\%$ in the measurement of energy (T), using the energy equation with symmetrical detector situation on either side of the beam. However, for simultaneous cross-section measurement with this energy measurement, one can move the conjugate counter to a place near the calculated value of Θ_1 for a given centre of mass angle in which scattering cross-section is being determined and look for the exact position where coincidence rate becomes maximum.

Calculation for this purpose have been done with the help of CERN computer CDC6400, and authors are highly thankful to Dr. J. B. M. Pattison, group leader for the Neutrino Beam Study Group, and to Dr. V. M. Bhise, Reader in the department of applied mathematics of this Institute for continued interest and discussions.

References

Bhiday M R., Jennings R. E. & Kalamus P. I. P. 1958 *Proc. Phys. Soc.* **72**, 973.

Bhabha H. J. 1936 *Proc. Roy. Soc.* **A154**, 195.

Dougal R. C. 1965 *Proc. Phys. Soc.* **85**, 851.

Hušàk V. & Peřinovà V. 1969 *Phys. Med. Biol.* **14**, 233.

Koch H. W., Motz J. W. & Olsen H. 1964 *Rev. Mod. Phys.* **36**, 881.

Moller C. 1932 *Ann. der Phys.* **14**, 531.

Seigbahn K. 1965 *Alpha, Beta and Gamma-ray Spectroscopy—I*, 485, North Holland Publishing Co. Amsterdam.

Wilkinson D. H. 1952 *Rev. Sci. Instr.* **23**, 414.

Wiegand C. 1959 *Proc. Int. Conf. High. Energy. Acc & Instr.* CERN, 581.